**[Total No. of Questions - 9]** **[Total No. of Printed Pages - 4]**

**(2063)**

**843**

**B.Tech 4th Semester Examination**

**Probability/Statistics/Queuing Theory**

**AS-4001**

**Time : 3 Hours** **Max. Marks : 100**

*The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.*

**Note :** Candidates are required to attempt five questions in all selecting one question from each section A, B, C, D of the question paper and all the subparts of the questions in section E. Use of non-programmable calculator is allowed.

**SECTION - A**

1. (i) In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4 and 2 percent are defective bolts. One bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C? **(4)**

(ii) A random variable X has the following probability distribution:

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| P(x) | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

(a) Determine the value of a.

(b) What is the smallest value of x for which $P(X < x) > 0.5$? and

(c) Find out the distribution function of X? **(8)**

(iii) In four tosses of a coin, let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. By simple counting, derive the distribution of X and hence calculate the expected value of X. **(8)**

2. (i) If $f(x, y) = 2 - x - y \ (0 \leq x \leq 1, 0 \leq y \leq 1)$;
$= 0$ elsewhere, find

(a) The marginal probability functions

(b) The conditional probability functions

(c) Var (X) and Var (Y)

(d) The coefficient of correlation between x and y. **(4)**

(ii) Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the given equation will have real roots. **(8)**

(iii) Use Chebychev's Inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6. **(8)**

## SECTION - B

3. (i) After correcting 50 pages of the proof of a book, the proof reader finds that there are, on the average two errors per 5 pages. How many pages would one expect to find with 0,1,2,3 and 4 errors, in 1000 pages of the first print of the book? (Given that $e^{-0.4} = 0.6703$). **(4)**

(ii) Seven coins are tossed and number of heads noted. The experiment is repeated 128 times and the following distribution is obtained:

| No. of heads | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|---|
| Frequencies | 7 | 6 | 19 | 35 | 30 | 23 | 7 | 1 | 128 |

Fit a Binomial distribution assuming

(a) The coin is unbiased

(b) The nature of the coin is not known

(c) Probability of a head for four coins is 0.5 and for the remaining three coins is 0.45. **(8)**

(iii) Derive the following particular cases of g.p.s.d

(a) Binomial Distribution (b) Negative Binomial Distribution (c) Poisson Distribution **(8)**

4. (i) If X and Y are independent normal variates with means 6, 7 and variances 9, 16 respectively, determine $\lambda$ such that $P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda)$. **(4)**

(ii) Obtain the Moment Generating Function and Cumulative Function of the Binomial Distribution. **(8)**

(iii) Fit a Poisson distribution to the following data which gives the number of yeast cells per square for 400 squares.

| No. of cells per square (x) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. of squares (f) | 103 | 143 | 98 | 42 | 8 | 4 | 2 | 0 | 0 | 0 | 0 | 400 |

(Given that $e^{-1.32}=0.2674$). **(8)**

## SECTION - C

5. (i) The first of the two samples has 100 items with means 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, , find the standard deviation of second group. **(4)**

(ii) Find the mean, median and mode for the following distribution of 140 students obtaining marks X or higher in a certain examination. **(8)**

| X | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| c.f. | 140 | 133 | 118 | 100 | 75 | 45 | 25 | 9 | 2 | 0 |

(iii) Let $\bar{X}$, $\bar{Y}$, $\sigma_x^2$, $\sigma_y^2$, and r be the means, variances and correlations between X and Y. If the regression of Y on X is linear, then

$$E(Y / X) = \bar{Y} + r\frac{\sigma_y}{\sigma_x}(X - \bar{X})$$

Similarly, if the regression of X on Y is linear, then

$$E(X / Y) = \bar{X} + r\frac{\sigma_x}{\sigma_y}(Y - \bar{Y})$$

**(8)**

6. (i) The following table shows the distribution of 100 families according to their expenditure per week. Number of families corresponding to expenditure group Rs. (10-20) and Rs. (30-40) are missing from the table. The median and the mode are given to be Rs. 25 and Rs. 24 respectively. Calculate the missing frequencies and then arithmetic mean of the data: **(4)**

| Expenditure | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| No. of families | 14 | ? | 27 | ? | 15 |

(ii) The first four moments of a distribution about the value 4 of the variable are -1.5, 17 and 108. Find the moments about mean $\beta_1$ and $\beta_2$. Also find the moments about the origin. **(8)**

(iii) Variable X and Y have the joint p.d.f.

$$f(x, y) = \frac{1}{3}(x + y),\ 0 \le x \le 1,\ 0 \le y \le 2.$$

find:

(a) r (X, Y)

(b) The two lines of regression

(c) The two regression curves for the means. **(8)**

## SECTION - D

7. (i) Derive Pollaczek-Khinchine formula for the average number of customers in the M/G/l queueing system. **(10)**

(ii) B & K Groceries operates with three check-out counters. The manager uses the following schedule to determine the number of counters in operation, depending on the number of customers in store:

| No. of customers in store | 1 to 3 | 4 to 6 | More than 6 |
|---|---|---|---|
| No. of counters in operation | 1 | 2 | 3 |

Customers arrive in the counters area according to a Poisson distribution with a mean rate of 10 customers per hour. The average check-out time per customer is exponential; with mean 12 minutes. Determine the steady state probability $p_n$ of n customers in the check-out area. **(10)**

8. (i) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following:
   (a) The average number of births per year.
   (b) The probability that no birth will occur in any one day.
   (c) The probability of issuing 50 birth certificates in 3 hour given that 40 certificates were issued during the first 2 hour of the 3 hour period. **(10)**

   (ii) Define open Jackson network and discuss for the steady state solution. Also write down the traffic equations in open Jackson network. **(10)**

## SECTION - E
## [Compulsory]

9. (i) A number is chosen at random among the first 120 natural number. The probability of the number chosen being multiple of 5 or 15 is ______________

   (ii) Define probability density function.

   (iii) Write the applications of central limit theorem.

   (iv) The mean of 20 observations is 15. On checking it was found that two observations were wrongly copied as 3 and 6. If wrong observations are replaced by correct values 8 and 4, then the correct mean is___________

   (v) In a perfectly symmetrical distribution 50% of items are above 60 and 75% are below 75. Therefore, the coefficient of quartile deviation is __________ and coefficient of skewness is __________

   (vi) The coefficient of correlation between X and Y is 0.6. Their covariance is 4.8. The variance of X is 9. Then the S.D. of Y is_______

   (vii) Define Geometric distribution and Lack of Memory

   (viii) Consider an M/M/l queueing system. If $\lambda$=6 and $\mu$ = 8, find the probability of at least 10 customers in the system.

   (ix) Define birth and death process. Also give an example in each case.

   (x) Define transient and steady states in queueing theory.

**(10×2=20)**