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(2064)

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B. Tech 2nd Semester Examination

Applied Maths-II (O.S.)

AS-1006

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Candidates are required to attempt five question in all, selecting one question from each of the section A, B, C & D and all subparts of the questions in Section-E. Use of non-programmable calculator is allowed.

SECTION - A

1. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). (10)

   (b) Evaluate $\int_0^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right) dt$, where $\vec{r} = 2t^2\hat{i} + t\hat{j} - 3t^3\hat{k}$ (10)

2. (a) Verify Stokes theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane. (10)

   (b) Find the curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2} \right)$ on the curve $x^3 + y^3 = 3axy$ (10)

SECTION - B

3. (a) Expand the function f(x) = x sin x as a Fourier series in the interval $-\pi \le x \le \pi$. Deduce that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \_\_\_\_\_ = \frac{\pi - 2}{4}$$ (10)

(b) Find Inverse Laplace Transform of $\frac{1}{s^3(s^2 + a^2)}$ (10)

4. (a) Find Fourier series of x^2 in $(-\pi, \pi)$. Use Parseval's identity to prove that

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots\ldots\ldots$$ (10)

(b) Find the Laplace Transform of f(t); defined as

$$f(t) = \begin{cases} \frac{t}{T} & , \text{ when } 0 < t < T \\ 1 & , \text{ when } t > T \end{cases}$$ (10)

SECTION - C

5. (a) Solve in series the equation

$$\left(1 + x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$ (10)

(b) Prove that $\int xJ_0^2 dx = \frac{1}{2}x^2\left[J_0^2(x) + J_1^2(x)\right] + C,$ where C is a constant (10)

6. (a) Solve the Legendre's differential equation

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$ (10)

(b) Prove that $\int_{-1}^{1} xP_n(x)P_n'(x)dx = \frac{2n}{2n+1}$ (10)

SECTION - D

7. (a) Form a partial differential equation by eliminating the arbitrary functions from the relation

$$z = f\left(\frac{x}{y}\right) + g(xy)$$ (10)

(b) Solve, $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$ for $0 < x < \pi, 0 < y < \pi$ given that

$$u(0, y) = u(\pi, y) = u(x, \pi) = 0, \; u(x, 0) = \sin^2 x$$ (10)

8. (a) Solve the equation:

$$\frac{du}{dx} = \frac{2du}{dt} + u, \quad \text{given} \quad u(x, 0) = 6e^{-3x}$$ (10)

(b) An insulated rod of length l has its ends A and B maintained at 0ºC and 100ºC respectively until steady state condition prevail. If B is suddenly reduced to 0ºC and maintained at 0ºC, find the temp. at a distance x from A at a time 't'. (10)

SECTION - E

9. (a) Verify that $e^{-k^2 t}.\sin\left(\frac{kx}{c}\right)$ is a solution of the heat equation

$$\frac{du}{dt} = c^2 \frac{d^2u}{dx^2}$$

(b) Form partial differential equation by eliminating arbitrary constants from the equation

$$z = (x^2 + \alpha)\,(y^2 + \beta)$$

(c) Find Laplace Transform of $t^2 \sin at$

[P.T.O.]

(d) Define Unit Impulse Function and write its Laplace Transform.

(e) Prove that $L^{-1}\left[\frac{c^{-\frac{1}{s}}}{s}\right] = J_0\left(2\sqrt{t}\right)$

(f) Obtain first three terms of the half-range sine series for e^x in $0 < x < 1$.

(g) Use divergence theorem to show that $\oint_c \nabla r^2 d\vec{s} = 6V$, where s is any closed surface enclosing volume V.

(h) Find curl $\vec{F}$ where $\vec{F} = \text{grad}\ (x^3 + y^3 + z^3 - 3xyz)$

(i) If $\hat{R}$ is a unit vector in the direction of $\vec{r}$, prove that $\hat{R} \times \frac{d\hat{R}}{dt} = \frac{\vec{r}}{r^2} \times \frac{d\vec{r}}{dt}$, where $r = |\vec{r}|$

(j) Derive formula for radius of curvature when the equations of the curve be x = f(s), y = g(s). (2×10=20)