14604

B. Tech 2nd Semester Examination
Engineering Mathematics-II (N.S.)

NS-104

Time : 3 Hours Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note: Candidates are required to attempt five questions in all selecting one question from each of the sections A, B, C and D of the question paper and all the subparts of the questions in section E. Use of non-programmable calculators are allowed.

SECTION - A

1. (a) Test the convergence of the series
\[ \frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots \ldots \] (10)

(b) Examine the convergence of the series
\[ \sum_{n=1}^{\infty} \frac{n^n}{n!} \] (10)

2. (a) Using integral test, show that the series
\[ \sum_{n=1}^{\infty} \frac{n^p}{n!}, p > 1 \] converges and its sum lies between \[ \frac{1}{p-1} \]
and \[ \frac{p}{p-1} \] (10)

(b) Prove that the series \[ x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \ldots \infty \] convergent for \(-1 < x \leq 1\). Also write the interval of convergence.

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[P.T.O.]
SECTION - B

3. (a) Find a Fourier series to represent \( x - x^2 \) from \( x = -\pi \) to \( x = \pi \). Hence show that:
\[
\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}
\] (10)

(b) Find the Fourier series to represent the function
\[
f(x) = \begin{cases} 
  a & \text{for } 0 < x < \pi \\
  -a & \text{for } \pi < x < 2\pi 
\end{cases}
\] (10)

4. (a) Expand \( \pi x - x^2 \) in a half range sine series in the interval \( (0, \pi) \) upto first three terms. (10)

(b) Find the Fourier series for the periodic function \( f(x) \) with period \( 2\pi \) defined by
\[
f(x) = \begin{cases} 
  0 & \text{for } -\pi < x \leq 0 \\
  \pi & \text{for } 0 < x < \pi 
\end{cases}
\]
What is the sum of series at \( x = 0, \pm \pi \)? (10)

SECTION - C

5. (a) Solve \( (xy^3 + y) \, dx + 2(x^2y^2 + x + y^4) \, dy = 0 \) (10)

(b) Solve \( y = 2px + y^2p^3 \) where \( p = \frac{dy}{dx} \) (10)

6. (a) Solve \( (x + 1)^2 \, \frac{d^2y}{dx^2} + (x + 1) \, \frac{dy}{dx} + y = \sin(2\log(x + 1)) \) (10)

(b) Solve \( \frac{d^3y}{dx^3} + 4y = 4\tan2x \) using variation of parameters. (10)

SECTION - D

7. (a) Prove that \( \text{div} \ (A \times B) = B \cdot \text{curl} \ A - A \cdot \text{curl} \ B \). where \( A \) and \( B \) are differentiable vectors function. (10)
(b) Evaluate \( \oint_C (yz\,dx + xz\,dy + xy\,dz) \) by Stoke's theorem where \( C \) is the curve \( x^2 + y^2 = 1, z = y^2 \). \hspace{1cm} (10)

8. (a) Using Green theorem, evaluate
\[ \oint_C \left[ (\cos x \sin y - xy)\,dx + \sin x \cos y\,dy \right], \] where \( C \) is the circle \( x^2 + y^2 = 1 \). \hspace{1cm} (10)

(b) Show that the vector \( V = (\sin y + z)\hat{i} + (x \cos y - z)\hat{j} + (x - y)\hat{k} \) is irrotational. \hspace{1cm} (10)

SECTION - E

9. (a) If \( \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \), prove that \( \text{curl} \vec{r} = 0 \).

(b) Define solenoidal vector.

(c) Define convergent series.

(d) State Gauss's test of infinite series.

(e) Find the particular integral of
\[ \frac{d^2y}{dx^2} + 4y = \sin 2x. \]

(f) Solve \( p = \tan (xp-y) \), where \( p = \frac{dy}{dx} \)

(g) State Dirichlet's condition for convergence of Fourier series.

(h) Find the Fourier coefficients for \( f(x) = |x| (-\pi < x < \pi) \)

(i) If a series \( \sum u_n \) is convergent, show that \( \lim_{n \to \infty} u_n = 0 \)

(j) Find the integrating factor of the differential equation \( (y-1)\,dx - x\,dy = 0 \) so that it becomes exact. \hspace{1cm} (10 \times 2 = 20)