(2064)

14604

B. Tech 2nd Semester Examination

Engineering Mathematics-II (N.S.)

NS-104

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Candidates are required to attempt five questions in all selecting one question from each of the sections A, B, C and D of the question paper and all the subparts of the questions in section E. Use of non-programmable calculators are allowed.

SECTION - A

1. (a) Test the convergence of the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\ldots. \quad (10)$$

(b) Examine the convergence of the series

$$\sum \frac{n^n.x^n}{n!} \quad (10)$$

2. (a) Using integral test, show that the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, p > 1 converges and its sum lies between $\frac{1}{p-1}$ and $\frac{p}{p-1}$ (10)

(b) Prove that the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\ldots\infty$ convergent for $-1 < x \leq 1$. Also write the interval of convergence. (10)

SECTION - B

3. (a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that :

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \text{..........} = \frac{\pi^2}{12}$$ (10)

(b) Find the Fourier series to represent the function

$$f(x) = \begin{cases} a & \text{for } 0 < x < \pi \\ -a & \text{for } \pi < x < 2\pi \end{cases}$$ (10)

4. (a) Expand $\pi x - x^2$ in a half range sine series in the interval $(0, \pi)$ upto first three terms. (10)

(b) Find the Fourier series for the periodic function f(x) with period 2π defined by $f(x) = \begin{cases} 0 & , \quad -\pi < x \le 0 \\ \pi & \quad 0 \le x < \pi \end{cases}$

What is the sum of series at $x = 0, \pm\pi$? (10)

SECTION - C

5. (a) Solve $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)dy = 0$ (10)

(b) Solve $y = 2px + y^2p^3$ where $p = \frac{dy}{dx}$ (10)

6. (a) Solve $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = \sin(2\log(x+1))$ (10)

(b) Solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ using variation of parameters. (10)

SECTION - D

7. (a) Prove that div (A×B)=B.curl A–A. curl B. where A and B are differentiable vectors function. (10)

(b) Evaluate $\oint_c (yzdx + xzdy + xydz)$ by Stoke's theorem where C is the curve $x^2 + y^2 = 1, z = y^2$. (10)

8. (a) Using Green theorem, evaluate $\oint_c [(\cos x \sin y - xy) dx + \sin x \cos y dy]$, where C is the circle $x^2 + y^2 = 1$. (10)

(b) Show that the vector $V = (\sin y + z)\hat{i} + (x \cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. (10)

SECTION - E

9. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that curl $\vec{r} = 0$.

(b) Define solenoidal vector.

(c) Define convergent series.

(d) State Gauss's test of infinite series.

(e) Find the particular integral of

$$\frac{d^2y}{dx^2} + 4y = \sin 2x.$$

(f) Solve p = tan (xp–y), where $p = \frac{dy}{dx}$

(g) Sate Drichlet's condition for convergence of Fourier series.

(h) Find the Fourier coefficients for $f(x) = |x| \ (-\pi < x < \pi)$

(i) If a series $\sum u_n$ is convergent, show that $\underset{n \to \infty}{Lt} u_n = 0$

(j) Find the integrating factor of the differential equation (y–1) dx – xdy = 0 so that it becomes exact. (10×2=20)