[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2064)

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MCA 1st Semester Examination

Mathematics (N.S.)

MCA-104

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each of the sections A, B, C & D, and all the subparts of the question in Section E. Use of scientific calculators is not allowed. All questions carry equal marks.

SECTION - A

1. (a) Show that the set D_{20} of all the positive divisors of 20 form a lattice with the relation of divisibility. Also represent the lattice D_{20} by a Hasse diagram. (6)

   (b) Simplify the following Boolean expression

   [a(a+b)+(b'+a)b]' (6)

2. (a) The probability distribution of a random variable x is: $f(x) = k \sin \frac{nx}{5}, 0 \le x \le 5.$ Determine the constant k and obtain the median. (6)

   (b) Construct an input/output table for the Boolean function defined by f(a, b) = (a.b)' + b. (6)

SECTION - B

3. (a) Show that the roots of the equation $z^2 + \alpha z + \beta = 0$, where α, β are complex numbers, are real if

$$(\bar{\alpha} - a)\left(\alpha\bar{\beta} - \bar{\alpha}\beta\right) = \left(\beta - \bar{\beta}\right)^2 .$$ (6)

(b) If $z_k = \cos\frac{\pi}{2^k} + i\sin\frac{\pi}{2^k}$, $k = 1, 2, 3, \ldots..$ then show that

$$z_1 z_2 z_3 \ldots..\infty = -1$$ (6)

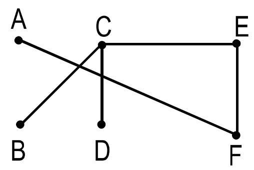
4. (a) Consider the following graph and find the followings:

(i) All simple paths from B to F.

(ii) All cut points.

(iii) All cycles.

(iv) All brides.



(v) Subgraph H of G generated by V' = {B, C, D, E}. (6)

(b) Show that V – E + R = 2 for any connected planer graph, where V, E and R are the number of vertices, edges and regions of the graph respectively. (6)

SECTION - C

5. (a) Consider the system kx+y+z=1, x+ky+z=1, x+y+kz=1. Use determinants to find those values of k for which the system has a unique solution. (6)

(b) Suppose $v_1, v_2, \ldots, v_n$ are non zero eigen vectors of T belonging to distinct eigen values $\lambda_1, \lambda_2 \ldots, \lambda_n$. Show that $v_1, v_2, \ldots, v_n$ are L.I. (6)

6. (a) If $x^x y^y z^z = c$, show that at $x=y=z$, $\frac{\partial^2 z}{\partial x \partial y} = (x \log ex)^{-1}$. (6)

(b) Check the convergence of $\int_{-\infty}^{\infty} xe^{-x^2} dx$. (6)

SECTION - D

7. (a) Apply Newton-Raphson method to determine the root of the equation $f(x) = x^3 - 5x + 1 = 0$. (6)

(b) Find the remainder of the Simpson's 3/8 rule

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] \text{ for}$$

equally spaced points $x_i = x_0 + ih$, $i = 1, 2, 3$. (6)

8. (a) Discuss the 'secant method' for the solution of non linear algebraic equations along with its geometrical significance. (6)

(c) Solve the equations

$x_1 + x_2 + x_3 = 6, \quad 3x_1 + (3+ \in)x_2 + 4x_3 = 20,$

$2x_1 + x_2 + x_3 = 13,$ using the Gauss elimination method,

where $\in$ is small such that $1 \pm \in^2 \approx 1$. (6)

SECTION - E

9. (a) Write the logical expression for the following sentence: "You can access the internet from campus only if you are a computer science major or you are not a freshman".

(b) Define a 'partial order relation'.

(c) If $z_1 = -3$ and $z_2 = 4i$, show that $\text{Arg}(z_1 z_2) \neq \text{Arg}(z_1) + \text{Arg}(z_2)$.

[P.T.O.]

(d) In how many ways can two adjacent squares be selected from an 8×8 chessboard?

(e) Find x and B if $B = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric.

(f) Evaluate $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\,dx.$

(g) Write the geometrical significance of bisection method.

(h) A single letter is selected at random from the word 'probability'. Find the probability that it is a vowel.

(i) Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$

(j) Find both the maximum and minimum values of $x^3 - 2x^2 - 2x - 4$ on the interval [0, 3]. (1.2×10=12)