14622

B. Tech 4th Semester Examination
Numerical Methods for Engineers (N.S.)
NS-207

Time : 3 Hours Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all selecting one question from each of sections A, B, C and D. Question 9 in Section E is compulsory. All questions carry equal marks.

SECTION - A

1. (a) Find a root of an equation \( x^3 + 2x - 2 = 0 \) by using iteration method, correct up to three decimal places.
   (b) Discuss Newton Raphson method. Also show that the rate of convergence of Newton Raphson method is quadratic. (10+10=20)

2. (a) Solve the following system of equations by Gauss-Jordan elimination method
   \[ x_1 + x_2 + x_3 = 3; \quad 2x_1 + 3x_2 + x_3 = 6; \quad x_1 - x_2 - x_3 = -3. \]
   (b) Solve the following system of linear equations by Jacobi’s method correct up to four decimal places:
   \[ 27x + 6y - z = 54; \quad 6x + 15y + 2z = 72; \quad x + y + 54z = 110. \] (10+10=20)

SECTION - B

3. (a) If \( \pi \) is approximated as 3.14 instead of 3.1456, find the absolute, relative and percentage error. Also explain when relative error is a better indicator of the accuracy of a computer than the absolute error.

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(b) For the following table, find the interpolation polynomial using Lagrange's formula:

| x   |  0  |  2  |  4  |  8  |  | (10+10=20) |
|-----|-----|-----|-----|-----| |           |
| f(x)|  3  |  8  | 11  | 18  | |           |

4. (a) From the following table of values of x and f(x) determine the value of f(0.29) using Newton’s backward interpolation formula.

| x   | 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.30 |  | (10+10=20) |
|-----|------|------|------|------|------|------| |           |
| f(x)| 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 | |           |

(b) Use the central difference interpolation formula of Bessel to find the value of y at (i) x = 1.40 and (ii) x = 1.60 from the following table:

| X   | 1.0  | 1.25 | 1.5  | 1.75 | 2.0  |  | (10+10=20) |
|-----|------|------|------|------|------| |           |
| y   | 1.0000 | 1.0772 | 1.1447 | 1.2051 | 2.2599 | |           |

SECTION - C

5. (a) From the following table find the value of \(\frac{dy}{dx}\) and \(\frac{d^2y}{dx^2}\) at the point x = 1.5.

| X   | 1.5  | 2.0  | 2.5  | 3.0  | 3.5  | 4.0  |  | (10+10=20) |
|-----|------|------|------|------|------|------| |           |
| Y   | 3.375 | 7    | 13.625 | 24   | 38.875 | 59   | |           |

(b) Use Newton's interpolation formula to find y when x = 1.85 and 2.4 from the table:

| x   | 1.7  | 1.8  | 1.9  | 2.0  | 2.1  | 2.2  | 2.3  |  | (10+10=20) |
|-----|------|------|------|------|------|------|------| |           |

6. (a) Evaluate the integral \(\int_0^1 \frac{1}{1+x^2} \, dx\) using Simpson's 1/3 rule. Also compare the result with exact value.
(b) Use Newton's divided difference formula. Define \( f(x) \) as a polynomial in \( x \) from the following data:

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>(0)</th>
<th>(3)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3</td>
<td>(-6)</td>
<td>39</td>
<td>822</td>
<td>1611</td>
</tr>
</tbody>
</table>

(10+10=20)

SECTION - D

7. (a) Given \( y' = x^2 - y, \ y(0) = 1 \). Find \( y(0.1), \ y(0.2) \) using Runge-Kutta method of (i) second order, (ii) 4th order.

(b) Solve by Predictor-Corrector method, the differential equation \( \frac{dy}{dx} = \frac{1}{x+y}, \ y(0) = 2 \) at \( x = 0.8 \), given that \( y(0.2) = 2.0933, \ y(0.4) = 2.1755, \ y(0.6) = 2.2493 \)

8. (a) Use Taylor series method to obtain approximate value of \( y \) at \( x=0.2 \) for the differential equation \( \frac{dy}{dx} = 2y + 3e^x, \ y(0) = 0. \)

(b) Using Euler's method, solve for \( y \) at \( x=0.1 \) from \( \frac{dy}{dx} = x + y + xy, \ y(0) = 1 \) taking step size \( h=0.025 \).

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(10+10=20)

SECTION - E

9. (a) State intermediate value theorem.

(b) Write the Newton-Cotes Quadrature formula.

(c) Explain the rate of convergence.

(d) What are the limitations of iterative methods?

(e) Explain partial and complete pivoting.

(f) Give the difference between Regula-Falsi and Secant method.

(g) Write a short note on Romberg integration.

(h) Give the advantage of Bisection method.

(i) Prove that (i) \( \Delta = EV = VE = \delta E^{\frac{1}{2}} \) (ii) \( E = 1 + \Delta = e^{\delta} \)

(j) Define absolute, relative and percentage errors.

(10×2=20)