14666

B. Tech 4th Semester Examination
Numerical methods and Computer Programming (O.S.)
AS(ID)-4001

Time : 3 Hours  Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt Five question in all selecting one question from each of the section A, B, C, and D. Section E is compulsory, attempt all the subparts of this section.

SECTION - A

1. (a) Using Lagrange's Interpolation formula find \( f(a) \), given

\[
\begin{array}{cccc}
 x & 5 & 7 & 11 & 13 \\
 f(x) & 150 & 392 & 1452 & 2366 \\
\end{array}
\]

(b) By means of Newton's divided difference formula, find the value of \( f(8) \) and \( f(15) \) from the table

\[
\begin{array}{cccccccc}
 x & 4 & 5 & 7 & 10 & 11 & 13 \\
 f(x) & 48 & 100 & 294 & 900 & 1210 & 2028 \\
\end{array}
\]

2. (a) Interpolate the population of a town for the year 1974 using Gauss's backward formula, given that

Population (in thousand) : 12 15 20 27 39 52

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(b) Write a computer program in C/C++ for the Newton’s forward Interpolation method. (10)

SECTION - B

3. (a) Using the Bisection Method, Compute a root of \(x^3-x-1=0\) upto three decimal places in four stages. (10)

(b) Apply Newton’s Raphson method to find a real root of the equation \(3x = \cos x + 1\) (10)

4. (a) Solve the system of equations

\[
\begin{align*}
27x + 6y - z &= 85 \\
6x + 15y + 2z &= 72 \\
x + y + 54z &= 110
\end{align*}
\]

Using Gauss-Seidal method. (10)

(b) Using Relaxation method, solve the system of equations

\[
\begin{align*}
10x - 2y - 3z &= 205 \\
-2x + 10y - 2z &= 154 \\
-2x - y + 10z &= 120
\end{align*}
\]

SECTION - C

5. (a) Evaluate \(\int_0^6 \frac{dx}{1+x^2}\) using Simpson’s 1/3 and Weddle’s rules. (10)

(b) Evaluate \(\int_0^1 \frac{dx}{1+x^2}\) using Romberg’s Integration in two steps taking \(h = 0.5\) (10)
6. (a) Evaluate \( \frac{dy}{dx} \) at \( x = 1.5 \)

\[
\begin{array}{cccccc}
  x & 0 & .5 & 1 & 1.5 & 2 \\
  y & 0.3989 & 0.3521 & 0.2420 & 0.1295 & 0.0540
\end{array}
\]

(b) Write a computer program in C to approximate definite integral with Simpson's 1/3 Rule.

7. (a) Solve \( \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \) in \( 0 < x < 5, \ t \geq 0 \) given that \( u(x, 0) = 20, \ u(0, t) = 0, \ u(5, t) = 100. \) Compute \( u \) for the time-step with \( h = 1 \) by Crank-Nicholson's Method.

(b) Write the finite difference approximation to solve partial differential equation.

8. (a) Solve the Laplace equation over the square region, satisfying the boundary conditions.

\[
\begin{align*}
  u (0, y) &= 0, \ 0 \leq y \leq 3 \\
  u (3, y) &= 9 + y, \ 0 \leq y \leq 3 \\
  u (x, 0) &= 3x, \ 0 \leq x \leq 3 \\
  u (x, 3) &= 4x, \ 0 \leq x \leq 3
\end{align*}
\]

(b) Solve \( \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \) under conditions

\[
\begin{align*}
  u (0, t) &= u (1, t) = 0, \text{ and} \\
  u (x, 0) &= \sin \pi x, \ 0 \leq x \leq 1 \text{ using} \\
  \text{Schmidt method, by taking } h = 0.2 \text{ and } k = 0.02
\end{align*}
\]

[P.T.O.]
9. (a) If \( f(x) = \frac{1}{x^2} \), find \( f(a, b) \) and \( f(a, b, c) \) by using divided difference.

(b) Find the polynomial which takes the values:
\[
\begin{array}{ccc}
x & 0 & 1 & 2 \\
y & 1 & 2 & 1 \\
\end{array}
\]

(c) State the iterative formula for Regula Falsi method to solve \( f(x) = 0 \).

(d) Write the iterative formula of Newton's Raphson Method to find \( \sqrt{N} \).

(e) Write a sufficient condition for Gauss-Seidal method to converge.

(f) State Romberg's method Integration formula to find the value of \( I = \int_{a}^{b} f(x) dx \).

(g) Write a computer program in C for Newton's Raphson method.

(h) Write down the standard five point formula to solve \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \).

(i) Find \( \frac{dy}{dx} \) at \( x = 10 \), for the following data:
\[
\begin{array}{cccccc}
x & 2 & 4 & 6 & 8 & 10 \\
y & 6 & 54 & 134 & 246 & 390 \\
\end{array}
\]

(j) Evaluate \( \left( \frac{\Delta^2}{E} \right)^{10} \left( \frac{Ee^*}{\Delta^2e^*} \right) \) \( (2 \times 10 = 20) \)