[Total No. of Questions - 9] [Total No. of Printed Pages - 4] (2064)

14686

B. Tech 4th Semester Examination

Probability/Statistics/Queuing Theory (O.S.)

AS-4001

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Candidates are required to attempt five questions in all selecting one question from each section A, B, C, D of the question paper and all the subparts of the questions in section E. Use of non-programmable calculators is allowed.

SECTION - A

1. (i) A problem in statistics is given to the three students A, B and C whose chances of solving it are 1/ 2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently? (4)

   (ii) A man with n keys wants to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials required to open the door

   (a) if unsuccessful keys are not eliminated from further selection, and

   (b) if keys are eliminated from further selection. (8)

   (iii) The joint frequency function of (X, Y) is given by

   $f(x, y) = A\, e^{-x-y}(0 \leq x \leq y, 0 \leq y < + \infty)$;

   = 0 elsewhere, find

   (a) Determine A

   (b) The marginal probability functions of X and Y

   (c) Examine if X and Y are independent

   (d) The conditional density function of Y, when it is given X =2. (8)

2. (i) State and prove Chebychev's Inequality. (4)

(ii) Two fair dice are thrown independently. Three events A, B and C are defined as follows:
A: Odd face with first dice
B: Odd face with second dice
C: Sum of points on dice is odd
Are the events A, B and C mutually independent? (8)

(iii) The joint density function of X and Y is given by
$f(x, y) = \frac{1}{2} x e^{-y} (0 < x < 2, y > 0)$;
= 0 elsewhere.
Find the distribution of X + Y. (8)

SECTION - B

3. (i) An item is produced in large numbers. The machine is known to produce 5% defective. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are to be examined in order to get 2 defective? (4)

(ii) Find the Moment Generating Function of the binomial distribution and hence find its mean and variance. (8)

(iii) If X, Y are independent normal variates with means 6, 7 and variances 9,16 respectively, determine λ such that
$P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda)$. (8)

4. (i) If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively, find the probability that
(a) X + Y = K and (b) X = Y. (4)

(ii) Fit a negative binomial distribution and calculate the expected frequencies for the following distribution:

No. of cells(x)	0	1	2	3	4	5	Total
Frequency(f)	213	128	37	18	3	1	400

(8)

(iii) Define Power Series Distribution and find its Moment Generating Function. Also find the Recurrence Relation for Cumulants of p.s.d. (8)

SECTION - C

5. (i) Find the median, lower and upper quartile, 4th decile and 40th percentile for the following distribution.

Marks	0-4	4-8	8-12	12-14	14-18	18-20	20-25	25 and over
No. of students	10	12	18	7	5	8	4	6

(4)

(ii) An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results:

	Firm A	Firm B
No. of workers	500	600
Average monthly wages	Rs. 186	Rs. 175
Variances of distribution of wages	81	100

Find

(a) Which firm, A or B, has a larger wage bill?

(b) In which firm A or B, is there greater variability in individual wages?

(c) Also calculate the average monthly wages and the variances of the two firms taken together. (8)

(iii) The joint density function of X and Y is given by

$f(x, y) = x + y, 0 < x < 1, 0 < y < 1$

$= 0$, otherwise

Find:

(a) The correlation coefficient between X and Y

(b) The regression curve of Y on X

(c) The regression curve of X on Y (8)

6. (i) The median and mode of the following wage distribution are known to be Rs. 33.50 and Rs. 34 respectively. Find the missing frequencies and then the arithmetic mean from the following data:

Wages (in Rs.)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	16	?	?	?	6	4

(4)

(ii) The lines of regression in a bivariate distribution are:

$X + 9Y = 7$ and $Y + 4X = 49/3$.

Find

(a) The coefficient of correlation,

(b) The ratios $\sigma_2^x : \sigma_y^2 : \text{Cov}\ (X, Y)$,

(c) The means of the distribution and,

(d) E(X/Y=1). (8)

(iii) Derive the formula which expresses the multiple correlation coefficients in terms of the total correlation coefficients between the pairs of the variables. (8)

[P.T.O.]

SECTION - D

7. (i) Discuss the pure birth process and hence obtain its probabilities, mean and variance. (10)

   (ii) The florist section in a grocery store stocks 18 dozen roses at the beginning of each week. On the average the florist sells 3 dozens a day, but the actual demand follows a Poisson distribution. Whenever the stock level reaches 5 dozens, a new order of 18 dozens is placed for delivery at the beginning of the following week. Because of the nature of the item, all roses left at the end of the week are disposed of. Determine the following:
   (a) The probability of placing an order in any one day of the week.
   (b) The average number of dozen roses that will be discarded at the end of the week. (10)

8. (i) Derive Pollaczek- Khinchine formula for the average number of customers in the M/G/l queueing system. (10)

   (ii) An operator attends five automatic machines. After each machine completes a batch run, the operator must reset it before a new batch is started. The time to complete a batch run is exponential with mean 45 minutes. The set time is also exponential with mean 8 minutes. Find
   (a) The average number of machines that are awaiting set up or are being set up.
   (b) The probability that all machines are working.
   (c) The average time a machine is down. (10)

SECTION - E

9. (i) The mean and median of 100 items are 50 and 52 respectively. The value of the largest items is 100. It was later found that it is actually 110. Therefore, the true mean is.....and the true median is.....

   (ii) Find the mean and the variance of the distribution in which the values of X are 1, 2, 3.....n, the frequency of each being unity.

   (iii) Define multiple and partial correlation.

   (iv) Normal distribution is a limiting form of the binomial distribution under the condition (a) .....and (b).....

   (v) The probability mass function of a negative binomial distribution is given by.....

   (vi) Write the moment generating function of a geometric distribution.

   (vii) Distinguish between correlation and regression.

   (viii) What are the basic characteristics of a queueing system?.

   (ix) Write the traffic equations in open Jackson networks.

   (x) Define transient and steady states in queueing theory.

(10×2=20)