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M. Tech 3rd Semester Examination

Advanced Mathematics

EC-306

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt any five questions.

1. (a) Find a power series solution in powers of x of the following differential equation

$$y'' - 3y' + 2y = 0 \quad (8)$$

- (b) Express $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial. (4)

(c) Show that $\int_{-1}^1 (1-x^2) [P_n'(x)]^2 dx = \frac{2n(n+1)}{2n+1}$ (8)

2. (a) Find the basic solution of the differential equation by Frobenius method:

$$(x-1)^2 y'' + (x-1)y' - 4y = 0 \quad (10)$$

- (b) Prove that :

$$\frac{d}{dx} [J_n^2(x)] = \frac{x}{2n} \{J_{n-1}^2(x) - J_{n+1}^2(x)\} \quad (10)$$

3. (a) Obtain the solution of given equation in terms of Bessel function:

$$xy'' + 2y' + \frac{1}{2}xy = 0 \quad (10)$$

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- (b) Find the eigen values and eigen functions of the given problem.

$$(x^{-1}y')' + (\lambda + 1)x^{-3}y = 0$$

$$y(1) = 0, y(e) = 0$$

(10)

4. (a) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$. Also find the image of the line $y = mx$ under this transformation. (10)

- (b) Reduce the quadratic form

$$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy \text{ to the canonical form.}$$

(10)

5. (a) If α and β are imaginary cube roots of unity, prove that

$$\alpha e^{\alpha x} + \beta e^{\beta x} = -e^{-x/2} \left(\cos \frac{\sqrt{3}}{2} x + \sin \frac{\sqrt{3}}{2} x \right) \quad (7)$$

- (b) If $\cos^{-1}(u + iv) = A + iB$, Prove that $\cos^2 \alpha$ and $\cosh^2 \beta$ are the roots of the equation:

$$x^2 - (1 + u^2 + v^2)x + u^2 = 0 \quad (7)$$

- (c) Prove that $\text{Log}_i i = \frac{4m + 1}{4n + 1}$, where m and n are integers. (6)

6. (a) Show that :

(i) The eigen values of a skew Hermitian matrix are purely imaginary or zero.

(ii) A Hermitian matrix remains Hermitian when transformed by an orthogonal matrix. (10)

- (b) Solve the following system of equation by Gauss elimination method:

$$5x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 7x_2 + x_3 + x_4 = 12$$

$$x_1 + x_2 + 6x_3 + x_4 = -5$$

$$x_1 + x_2 + x_3 + 4x_4 = -6$$

(10)

7. (a) Using Crout's triangularization method, solve:
 $x + y + z = 1$
 $4x + 3y - z = 6$
 $3x + 5y + 3z = 4$ (10)
- (b) Solve the following system of equation by Jacobi's method:
 $5x + 2y + z = 12$
 $x + 4y + 2z = 15$
 $x + 2y + 5z = 20$ (10)
8. (a) Using any iteration method, find all the eigen values and the eigen vectors of the matrix
$$\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$
 (10)
- (b) What is meant by Monte-Carlo method of simulation? Discuss its scope and use in problems encountered in waiting line. (10)