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(2063)

801

B.Tech 2nd Semester Examination

Engineering Mathematics-II (NS)

NS-104

Time : 3 Hours **Max. Marks : 50**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/ continuation sheet will be issued.

Note : Attempt five questions in all selecting one question from each sections A, B, C and D of the question paper and all subparts of the question in section E. Use of non-programmable calculators are allowed.

SECTION - A

1. (a) Test the series for convergence.

$$\frac{2}{2.3.4}+\frac{4}{3.4.5}+\frac{6}{4.5.6}+\ldots\ldots\ldots \qquad \textbf{(5)}$$

(b) Discuss the convergence of series

$$\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2 x^2+\left(\frac{4}{3}\right)^3 x^3+\ldots\ldots\ldots\ldots\infty \qquad \textbf{(5)}$$

2. (a) Using Integral test, show that the series

$\sum ne^{-n^2}$ converges **(5)**

(b) Examine the convergence of the series

$$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4)+\ldots. \qquad \textbf{(5)}$$

SECTION - B

3. (a) Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0 < x < 2\pi$. **(5)**

(b) Expand in series of sines and cosines of multiple angles of x, the periodic function f(x) with period 2π defined as

$$f(x)=\begin{cases}-1 \text{ for } -\pi < x < 0\\ 1 \text{ for } 0 \le x \le \pi\end{cases}$$

Also calculate the sum of the series at $x=0, \pm \pi$. **(5)**

4. (a) Obtain the half range series for e^x in $0 < x < 1$. **(5)**

(b) If the Fourier series $\frac{a_0}{2}+\sum_{n=1}^{\infty}$ (a_n cos nx + b_n sin nx) of f(x) converges to f(x at every point of the closed interval $[0, 2\pi]$, then prove that

$$\frac{1}{\pi}\int_0^{2\pi}\left[(x)\right]^2 dx=\frac{a_0^2}{2}+\sum_{n-1}^{\infty}\left(a_n^2+b_n^2\right)$$ **(5)**

SECTION - C

5. (a) Solve $\frac{dy}{dx}+y\sec x+\tan x$ **(5)**

(b) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\ \sin(\log x)$ **(5)**

6. (a) Using method of variation of parameter solve $\frac{d^2y}{dx^2}+16y=32\sec x$ **(5)**

(b) Solve $x^2p^2 + xyp - 6y^2 = 0,$ where $b = \frac{dy}{dx}$ **(5)**

SECTION - D

7. (a) Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla - A) - \nabla^2 A,$ where ∇ is a vector differentiable operator and A is a differentiable vector function. **(5)**

(b) Evaluate by stoke's Theorem $\oint_C \left(e^x dx + 2ydy - dx\right)$ where C is the curve $x^2 + y^2 = 4, z = 2$. **(5)**

8. (a) Evaluate $\oint_C \left[\left(x^2 - \cosh y\right)dx + \left(y + \sin x\right)dy\right]$ using Green's the orem, where C is the rectangle with vertices (0, 0), (π, 0), (π, 1), (0, 1). **(5)**

(b) Evaluate $\iiint_V \phi dv,$ where, $\phi = 45x^2y$ and V is the closed region bounded by the planes $4x + 2yz = 8, x = 0, y = 0, z = 0.$ **(5)**

SECTION - E

9. (a) Show the series $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ is convergent and converges to 2. **(1)**

(b) Define power series and its interval of convergence. **(1)**

(c) Discus Fourier series for even and odd functions. **(1)**

(d) Expand in a Fourier series, the function f(x)=x in the interval $[-\pi, \pi]$ **(1)**

(e) Solve: $\frac{d^4y}{dx^4} + 4x = 0$ **(1)**

(f) Define Clairaut equation and also find its solution. **(1)**

(g) If $\vec{r} = x\hat{i} + y\hat{j} + 3\hat{k}$, show that $\text{div}\,\vec{r} = 3$ **(1)**

(h) Define irrotational vectors field. **(1)**

(i) Find the particular integral of

$$\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = e^{2x}$$ **(1)**

(j) State Leibnitz's Rule for the convergence of an alternating infinite series. **(1)**