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M. Tech 1st Semester Examination
Advanced Mathematics
EE1-513

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt any Five Questions. All questions carry equal marks. Use of non-programmable calculator is allowed.

1. (a) Evaluate Laplace transform of the function

$$f(t) = 2t e^{-3t} \sin t + t^2 \cos(3t) + e^{4t} \cos h(5t)$$

- (b) Evaluate inverse Laplace transform of the function

$$F(s) = \frac{2s^2 - 4}{(s+1)(s-2)(s-3)} \quad (20)$$

2. (a) Find Z-transform of the sequence

$$\{x_k\} = \{ka^{k-1}\}, |z| > a, a \text{ is constant}$$

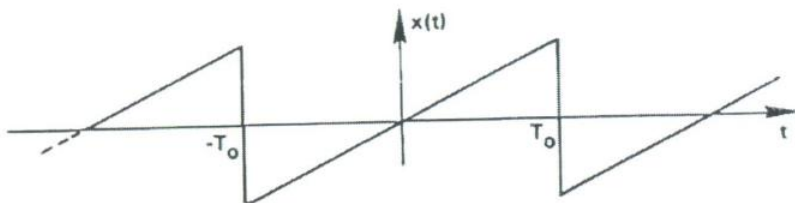
- (b) Find inverse Z-transform of $Y(z) = \frac{2z+1}{(z+1)(z-3)}$ (20)

3. (a) Determine convolution $h(t)*g(t)$, where $h(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$\text{and } g(t) = \begin{cases} \sin(t), & 0 \leq t \leq \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$

[P.T.O.]

- (b) Determine the Fourier series of the periodic function defined in the following figure. (20)



4. (a) Compute the discrete Fourier transform of $x(k - 2)$.
 (b) Demonstrate the frequency convolution theorem. (20)
5. (a) Compute Fast Fourier Transform for the function $h(t) = te^{-t}$, $t > 0$.
 (b) Find the two-dimensional Fourier transform of the function $h(x, y) = \cos(2\pi u_0 x) \cos(2\pi v_0 y)$ (20)
6. (a) If a circuit contains a resistance R , inductance L and capacitance C , is driven a potential of electromotive force $E(t) = 17 \sin(2t)$ volts. At time zero the current is zero and the charge on the capacitor is $1/2000$ coulomb. Obtain the charge $q(t)$ on the capacitor for $t > 0$.
 (b) Find the solution of the system of linear differential equation

$$X' = AX \text{ where } A = \begin{pmatrix} -\frac{3}{10} & \frac{3}{10} \\ \frac{1}{5} & -\frac{2}{5} \end{pmatrix}. \text{ The initial conditions}$$

$$\text{are } X_1(0) = 150, X_2(0) = 50. \quad (20)$$

7. (a) Find solution of the difference equation

$$y_{k+2} + y_{k+1} - 2y_k = 1 \quad (k \geq 0), \quad y_0 = 0 \text{ and } y_1 = 1,$$

- (b) Define trajectory, phase portrait and phase plane for the system of differential equations

$$x'(t) = f\{x(t), y(t)\}, \quad y'(t) = g\{x(t), y(t)\}.$$

Find phase portrait of the system $x' = -9y$, $y' = -4x$ (20)

- (c) (a) Define discrete random variable. A random variable X has the following probability function :

$X:$	0	1	2	3	4	5	6	7
$p(X):$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find the value of k , $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$. Also determine the distribution function of X .

- (b) Define Binomial distribution. Obtain its mean and variance. (20)