

[Total No. of Questions - 8] [Total No. of Printed Pages - 3]  
(2124)

1632

**M. Tech 1st Semester Examination**  
**Digital Control Systems**  
**EE1-514**

**Time : 3 Hours**

**Max. Marks : 100**

*The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.*

**Note :** Total number of questions listed in the question paper is eight. Attempt any five questions. All questions carry equal marks. Precise answers with clarity will be appreciated and avoid the misuse of answer scripts.

1. (a) Draw a block diagram of digital control systems. Explain (S/H) (Sample and Hold) and transducer blocks in detail with more theoretical justifications. (10)  
(b) Compare digital and analog controllers as well as explain the superiority of digital control methods over the analog. (10)
2. (a) Write the relation between the z transform, discrete-time Fourier transform, Laplace transform using the bilinear transformation. (10)  
(b) Compute z transform of the following step sequences:  
(i)  $u(k), u(-k-1)$  (ii)  $b^{|n|}$ , where  $b > 0$ . (10)
3. (a) Explain linear time-varying continuous and discrete-time systems by using the state space interpretations and output equations. (10)  
(b) Find the transfer functions of zero-order hold and nth order hold circuits in the discrete-time setting. Write the usefulness of the both circuits in detail in the context of discrete-time signal constructions. (10)

[P.T.O.]

4. (a) Explain the Jury stability criterion for discrete-time linear time-invariant systems. Check the stability of the matrix

$$G = \begin{pmatrix} 0.1 & 0.1 & 0 \\ 0.3 & -0.1 & -0.2 \\ 0 & 0 & -0.3 \end{pmatrix}$$

where the matrix  $G$  is associated with the discrete-time linear time-variant system. (10)

- (b) Find the solution of linear time-invariant discrete-time state equation coupled with the output equation.

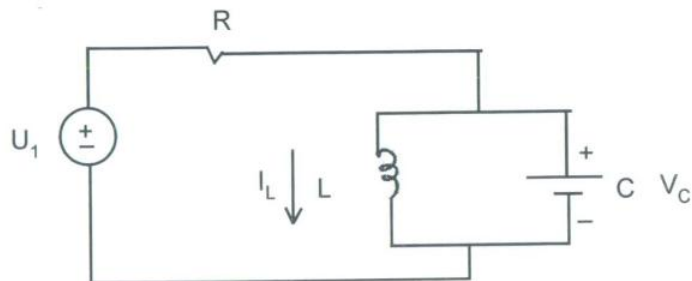
$$x(k+1) = Gx(k) + Hu(k),$$

$$y(k) = C^T x(k),$$

where  $u(k)$  is the control signal and  $y(k)$  is the output signal and  $x(k)$  is dimensional state vector. Making the use of the derived closed form solution, formulate the input-output relationship using the notion of convolution sum. (10)

5. (a) For the given linear time-invariant discrete-time state equation coupled with the output equation,  $x(k+1) = Gx(k) + Hu(k)$ ,  $y(k) = C^T x(k)$ , formulate the input-output relationship using the notion of convolution sum. (10)

- (b) Write down the state equation of the following electrical circuits in the continuous-time and discrete time setting (10)



6. (a) Explain controllability and observability of state equations and measurement equations in the discrete-time setting by stating their necessary and sufficient conditions. (10)

- (b) Consider the system defined by

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(k) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} v(k),$$

$$y(k) = (1 \ 0) \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix},$$

determine the conditions on  $a, b, c$  and  $d$  for complete state controllability and complete observability. (10)

7. The control system defined by

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -0.16 & -1 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} u(k), \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

examine whether the system is completely state controllable. If the system is controllable, determine a sequence of control signals  $u(0), u(1)$ , such that the  $x(2)$  becomes

$$\begin{pmatrix} x_1(2) \\ x_2(2) \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (20)$$

8. Obtain the state transition matrix of the following discrete-time system:

$$x(k+1) = Gx(k) + Hu(k),$$

$$y(k) = C^T x(k), \quad x(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

where  $G = \begin{pmatrix} 0 & 1 \\ -0.16 & -1 \end{pmatrix}$ ,  $H = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $C = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , then obtain the state

$x(k)$  and the output  $y(k)$  for the instants  $k = 1, 2$ , where the input  $u(k) = 1$  for  $k = 0, 1$ . (20)