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M. Tech 3rd Semester Examination

Advanced Mathematics

EC-306

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt any five questions.

1. (a) Use Frobenius method to solve the differential equation:
 $2xy'' + 3' - y = 0$ (10)
- (b) Prove that $2n J_n(x) = x [J_{n-1}(x) + J_{n+1}(x)]$ (5)
- (c) Show that $\frac{d}{dx}[J_n(x)] = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ (5)
2. (a) Convert parametric Bessel's equation
 $x^2y'' + xy' + (\lambda^2x^2 - m^2)y = 0$
in Sturm-Liouville form and explain why it is not a regular Sturm-Liouville problem. (4)
- (b) Expand the function $f(x) = x$ on $0 \leq x \leq 1$ using the eigen function of the system
 $y'' + \lambda y = 0$; given that $y'(0) = y(1) = 0$ (8)
- (c) Find the eigen values and eigen functions of Sturm-Liouville boundary value problem:
 $y''(x) + 4\lambda y(x) = 0, 0 < x < L$
given that $y(0) = y(L) = 0$ (8)
3. (a) Construct the bilinear transformation $\omega = s(z)$ that maps the points, $1, \pi, -\pi$ onto the points $0, 2, -2$ respectively. (10)

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- (b) Find the determinant of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 \\ 0 & 3 & 0 & 0 & 4 & 0 \\ 3 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Also find the rank of the matrix. (10)

4. (a) By using power series method, find the solution of the differential equation $y'' + \lambda y = 0$ in power of x . (8)
- (b) List first ten legendre's polynomials. (8)
- (c) If $P_n(x)$ is the legendre polynomial, then draw the graph of $P_5(x)$. (4)
5. (a) Whether the following matrix is positive definite or not, explain.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix} \quad (5)$$

- (b) Find the eigen values of the given matrix:

$$\begin{bmatrix} 1 & 5 & 6 & 7 \\ 0 & 2 & 9 & 8 \\ 0 & 0 & 7 & 11 \\ 0 & 0 & 0 & 20 \end{bmatrix} \quad (5)$$

- (c) Solve the following system of linear equations by Gauss elimination method:

$$5x_1 + x_2 + x_3 + x_4 = 6$$

$$x_1 + 6x_2 + x_3 + x_4 = 11$$

$$x_1 + x_2 + x_3 + 7x_4 = 24$$

$$x_1 + x_2 + 11x_3 + x_4 = 26$$

(10)

6. (a) Find principal value of $\log(2 + 3i)$. (6)
 (b) Let $x = (9 + 9i)^{1/6}$, then find all the values of x . (7)
 (c) Prove that:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (7)$$

7. (a) Write a note on system simulation and technique of simulation. (10)
 (b) Using any iteration method, find all the eigen values and eigen vectors of the matrix:

$$\begin{bmatrix} 6 & 0 & 9 \\ 0 & 2 & 3 \\ 9 & 3 & 5 \end{bmatrix} \quad (10)$$

8. (a) Solve the following system of linear equations by Crout's triangularization method:

$$\begin{aligned} 2x_1 - 2x_2 - 2x_3 &= -4 \\ -2x_2 + 2x_3 &= -2 \\ -x_1 + 5x_2 + 2x_3 &= 6 \end{aligned} \quad (10)$$

- (b) Solve the equations:

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

by Gauss-Seidal iteration method. (10)