

B. Tech 4th Semester Examination
Discrete Mathematics & Logic Design (OS)
IT-4003

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Each question carries 20 marks. Attempt one question from each section A, B, C & D. Section E is compulsory and carries 20 marks.

SECTION - A

- Define a Tautology and contradiction. For the propositions verify that
 $[(p \wedge q) \rightarrow r] \leftrightarrow [7(p \wedge q) \vee r]$ is a tautology using truth table.
 - Examine the validity of the following argument. "If prices are higher then wages are high. Prices are high or there are price controls. If there are price controls then there is not an inflation. There is an inflation therefore wages are high." (20)
- Define the following
 - Tautology
 - Contradiction
 - Contingency.
 - If x and y denote the pair of real numbers for which $0 < x < y$, prove by mathematical induction $0 < x^n < y^n$ for all natural number n . (20)

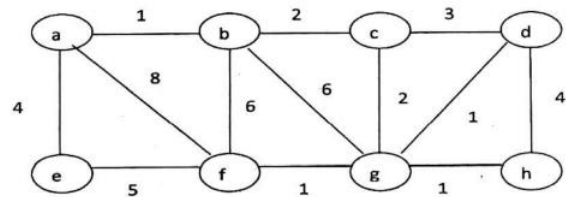
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SECTION - B

- Prove the theorem 'If x is an odd integer, x^2 is odd integer'.
 - If $R = \{(a, b), (b, c), (c, a)\}$ and $A = \{a, b, c\}$, then find reflexive, symmetric and transitive closure of R by the composition of relation R .
 - Prove that in a room of 13 people, 2 or more people have their birthdays in the same month. (20)
- If function f is one-one onto then inverse off i.e. f^{-1} is also one-one onto.
 - Show that $(\mathbb{Z}^+, \text{divisibility})$ is a poset.
 - How many selections any number at a time may be made from three white balls, four green balls, one red ball and one black ball if at least one must be chosen. (20)

SECTION - C

- Show the sum of the degrees of all the vertices in a graph is equal to twice the number of edges in the graph.
 - Prove that a graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. (20)
- Find out MST of graph in Fig. 1 using prims and kruskal algo. (20)



SECTION - D

7. (a) Define group, monoids, semigroups and subgroups.
(b) If G_1 and G_2 are two subgroups of a group G then prove that $G_1 \cap G_2$ is also a subgroup of G .
(c) For a Group G prove that G is abelian iff $(ab)^2 = a^2 b^2$ for all $a, b \in G$ (20)
8. (a) If H and K are any two subgroups of a group G , then show that $H \cup K$ will be a subgroup iff $H \subset K$ or $K \subset H$.
(b) Define field with one example.
(c) Consider a ring $(R, +, \cdot)$ defined by $a \cdot a = a$. Determine whether the ring is commutative or not. (20)

SECTION - E

9. Define all with the examples:
- (a) Euler path
(b) Set
(c) Function
(d) Permutation (4×5=20)