[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2125)

15013

B. Tech 1st Semester Examination

Engineering Mathematics-I (NS)

NS-101

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each section A, B, C & D of the question paper and all the subparts of the question in section E.

SECTION - A

1. (a) Investigate the values of λ and μ so that the equations

$$2x - 5y + 2z = 8,\ 2x + 4y + 6z = 5,\ x + 2y + \lambda z = \mu$$

have (i) unique solution (ii) no solution (iii) an infinite number of solutions.

(b) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(10+10=20)

2. (a) Reduce the quadratic form $2xy + 2xz - 2yz$ to a canonical form. Also write the model matrix and nature of the quadratic form.

(b) Define Unitary matrix. Show that the eigen values of a unitary matrix are of unit modulus. (10+10=20)

[P.T.O.]

SECTION - B

3. (a) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$$

(b) Find the maximum and minimum distance of the point (3,4,12) from the sphere

$x^2 + y^2 + z^2 = 1$ (10+10=20)

4. (a) If $x^2 + y^2 + z^2 - 2xyz = 1$. Show that

$$\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}} = 0$$

(b) Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum.

(10+10=20)

SECTION - C

5. (a) Evaluate $\iint_R (x+y)^2\, dxdy$, where R is the parallelogram in the xy-plane with vertices (1,0), (3,1), (2,2), (0,1) using the transformation $u = x + y$ and $v = x - 2y$.

(b) By transforming into cylindrical coordinates evaluate the integral $\iiint (x^2 + y^2 + z^2)\, dxdydz$ taken over the region $0 \le z \le x^2 + y^2 \le 1$. (10+10=20)

6. (a) Define Beta and Gama functions. Also derive the relation between them.

(b) Find the volume bounded by the paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$.

(10+10=20)

SECTION - D

7. (a) Use 'C+iS' method to find the sum of the following series

$$\frac{\sin\alpha}{1!} - \frac{\sin 2\alpha}{2!} + \frac{\sin 3\alpha}{3!} - \frac{\sin 4\alpha}{4!} + \ldots \infty.$$

(b) Expand $\sin^7\theta\cos^3\theta$ in a series of sines of multiples of θ.

(10+10=20)

8. (a) Separate $\tan^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts.

(b) If $\tan(\theta+i\phi)=\cos\alpha+i\sin\alpha$, prove that $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$ and

$\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$. (10+10=20)

SECTION - E

9. (a) If u is a homogenous function of degree n in x and y then prove that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

(b) If $x=r\cos\theta$, $y=r\sin\theta$,, $z = z$, evaluate $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$.

(c) Solve $\lim\limits_{(x,y)\to(0,0)} \frac{2xy}{(x^2+y^2)}$.

(d) Solve the integral $\int_0^\infty \frac{1}{\sqrt{1-x^4}}dx$ by using beta/gamma function.

(e) Separate tan (x+iy) into real and imaginary parts.

(f) Find the eigen values of $\begin{bmatrix} 1+i & -6 \\ 8 & 3-5i \end{bmatrix}$.

(g) Use Taylor's theorem to find $\sqrt{102}$.

(h) Find $\frac{dy}{dx}$, when $x^y+y^x=c$.

(i) Evaluate $\int_0^{\frac{\pi}{2}} \cos^4 3\theta \sin^3 6\theta \, d\theta$.

(j) Find the value of $\Gamma\left(\frac{1}{2}\right)$. (10×2=20)