[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
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B. Tech 1st Semester Examination

Engineering Mathematics-I (NS)

NS-101

Time : 3 Hours Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question each from section A, B, C & D of the question paper and all the subparts of the question in section E.

SECTION - A

1. (a) For what values of a and b do the equations x+2y+3z=6, x+3y+5z=9, 2x+5y+az=b have (i) no solution (ii) a unique solution (iii) more than one solution? (10)

   (b) Find the characteristic equation of the matrix $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$ and hence find the matrix represented by

   $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ (10)

2. (a) If λ be an eigen value of a non-singular matrix A, show that $\frac{|A|}{\lambda}$ is an eigen value of the matrix adj. A. (8)

   (b) Reduce $3x^2-2y^2-z^2-4xy+8xz+12yz$ into canonical form by orthogonal transformation. (12)

SECTION - B

3. (a) Prove that if $f(x,y)=\frac{1}{\sqrt{y}}.e^{\frac{-(x-a)^2}{4y}}$, then $f_{xy}=f_{yx}$. (10)

   (b) Verify Euler's theorem for $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$. (10)

4. (a) If $u=f(x^2+2yz, y^2+2zx)$, prove that

   $(y^2-zx)\frac{\partial u}{\partial x}+(x^2-yz)\frac{\partial u}{\partial y}+(z^2-xy)\frac{\partial u}{\partial z}=0$ (10)

   (b) Given x+y+z=a, find the maximum value of $x^m.y^n.z^p$. (10)

SECTION - C

5. (a) Change the order of integration in $\int_0^a\int_{x^2/a}^{2a-x} xy\,dy\,dx$ and hence evaluate the same. (10)

   (b) Find the volume of the tetrahedran bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (10)

6. (a) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (10)

   (b) Using triple integration, find the volume of the solid bounded by the surface $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}+\left(\frac{z}{c}\right)^{2/3}=1$. (10)

SECTION - D

7. (a) If $\tan(\alpha+i\beta)=x+iy$, then prove that

   (i) $x^2+y^2+2x\cot 2\alpha=1$ (3)

   (ii) $x^2+y^2-2y\coth 2\beta=-1$ (3)

   (iii) $x\sinh 2\beta=y\sin 2\alpha$ (4)

[P.T.O.]

(b) Sum the series

$$\frac{\cos\alpha}{\cos\alpha} + \frac{\cos 2\alpha}{2!\cos^2\alpha} + \frac{\cos 3\alpha}{3!\cos^2\alpha} + \text{-------}\infty \qquad (10)$$

8. (a) Prove that

$$[\cos\alpha - \cos\beta) + i(\sin\alpha - \sin\beta)]^n + [(\cos\alpha - \cos\beta) - i(\sin\alpha - \sin\beta)]^n$$

$$= 2^{n+1}\sin^n\left(\frac{\alpha-\beta}{2}\right).\cos\left(\frac{n(\pi+\alpha+\beta)}{2}\right) \qquad (10)$$

(b) Find all the values of $\left[\left(1+i\sqrt{3}\right)^{1/3} + \left(1-i\sqrt{3}\right)^{1/3}\right]$. (10)

SECTION - E

9. (a) Find the rank of the matrix : $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.

(b) Using Cayley-Hamilton theorem, find A^6 if

$$A = \begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$$

(c) Find the first order partial derivatives of

$$u = \frac{x}{y}\tan^{-1}\left(\frac{y}{x}\right)$$

(d) Expand $e^x \sin y$ in powers of x and y as far as terms of the third degree.

(e) If $r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\frac{y}{x}$, evaluate $\frac{\partial(r,\theta)}{\partial(x,y)}$.

(f) Evaluate the integral $\int_0^{\pi/2}\int_0^{a\cos\theta}\int_0^{\sqrt{a^2-r^2}} r\,dz\,dr\,d\theta$.

(g) Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^5 (\cos\theta - i\sin\theta)^3}{(\cos 5\theta + i\sin 5\theta)^7 (\cos 2\theta - i\sin 2\theta)^5}$.

(h) Separate cosech (x+iy) into real and imaginary parts.

(i) Prove that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist,

where $f(x, y) = \frac{x^2 y}{x^4+y^2}$, $(x, y) \neq (0,0)$.

(j) Are the following vectors linearly dependent? If so, find a relation between them.

$x_1 = (2,-1,4)$, $x_2 = (0,1,2)$, $x_3 = (6,-1,16)$. (2×10=20)