

SECTION - B

16024(J)  $\square-16$

B. Tech 2nd Semester Examination

Engineering Mathematics-II (NS)

NS-104

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each section A, B, C & D of the question paper and all the subparts of the question in section E. Use of non-programmable calculator is allowed.

SECTION - A

1. (a) Determine the nature of the following series

$$\frac{1}{x} + \frac{2!}{x(x+1)} + \frac{3!}{x(x+1)(x+2)} + \dots (x > 0)$$

- (b) Discuss the convergence of the series

$$\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \frac{5}{4^p} + \dots \infty \quad (10+10+20)$$

2. (a) Examine the behavior of the series  $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$

- (b) Test the following series for convergence and absolute convergence

$$1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \dots \infty \quad (10+10=20)$$

[P.T.O.]

3. (a) Find a Fourier series of the function  $f(x) = |\sin x|$  in the interval  $(-\pi, \pi)$

- (b) Obtain half-range cosine series of

$$f(x) = \begin{cases} kx & 0 \leq x \leq \frac{l}{2} \\ k(l-x) & \frac{l}{2} \leq x \leq l \end{cases}$$

Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty$

(10+10=20)

4. (a) Find the Fourier series of  $f(x) = x^2$  in  $-\pi < x < \pi$ . Use

Parseval's identity to prove that  $\frac{\pi^2}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots$

- (b) Obtain the Fourier Sine series for  $f(x) = 1$  in  $0 < x < \pi$  and hence show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots \quad (10+10=20)$$

SECTION - C

5. (a) Solve  $(\cos x + y \sin x) dx - (\cos x) dy = 0, y(\pi) = 0$

- (b) Using the method of variation of parameter solve

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \tan x \quad (10+10=20)$$

6. (a) Solve  $x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left( x + \frac{1}{x} \right)$

- (b) Solve  $xyp^2 + (3x^2 - 2y^2)p - 6xy = 0$ , where  $p = \frac{dy}{dx}$   
(10+10=20)

### SECTION - D

7. (a) Find the curvature and torsion of the curve  $x=acost$ ,  
 $y=asint, z=bt$ .
- (b) State Green's Theorem and evaluate  
 $\int_C (2xy - x^2)dx + (x^2 + y^2)dy$  where C is the boundary of  
the region enclosed by  $y = x^2$  and  $y^2=x$ . (10+10=20)
8. (a) If S is any closed surface enclosing a volume V and  
 $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$ , prove that  $\iint_S \vec{F} \cdot \hat{n} dS = 6V$ .
- (b) Evaluate  $\iint_S \vec{A} \cdot \hat{n} dS$ , where  $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$  and S is  
the surface of the cylinder  $x^2 + y^2 = 16$  included in the  
first octant between  $z = 0$  and  $z = 5$ . (10+10=20)

### SECTION - E

9. (i) If  $\vec{F}(t)$  has a constant magnitude then  $\vec{F} \cdot \frac{\partial \vec{F}}{\partial t} = 0$ .
- (ii) Show that  $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$
- (iii) Test whether the following series is absolutely convergent  
or conditionally convergent?  
 $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \infty$
- (iv) Solve  $(y - px)(p - 1) = p$  where  $p = \frac{dy}{dx}$ .

[P.T.O.]

- (v) Show that  $\frac{1}{D+a} X = e^{-ax} \int X e^{ax} dx$  where  $D = \frac{d}{dx}$ .
- (vi) Find the particular integral of  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = e^x$ .
- (vii) State Parseval's formula.
- (viii) Define positive term series and alternating series with  
example.
- (ix) Discuss Fourier series for even and odd function.
- (x) Give the physical significance of Curl. (10×2=20)