[Total No. of Questions - 9] **[Total No. of Printed Pages - 4]**
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B. Tech 1st Semester Examination

Applied Mathematics-I (O.S.)

AS-1001

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question each from Sections A, B, C and D. Q. No. 9 is compulsory.

SECTION - A

1. (a) State and prove Euler's Theorem. **(7½)**

   (b) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere

   $x^2 + y^2 + z^2 = 1.$ (Use Lagrange's method of multipliers). **(7½)**

2. (a) Find the volume (by double integrals) bounded by the cylinder $x^2 + y^2 = 4$ and the planes

   $y + z = 4$ and $z = 0$. **(7½)**

   (b) Evaluate the following integral by changing to spherical polar co-ordinates:

   $$\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_{\sqrt{(x^2+y^2)}}^1 \frac{dzdydx}{\sqrt{(x^2+y^2+z^2)}}.$$ **(7½)**

SECTION - B

3. (a) Find the length of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first quadrant. **(7½)**

   (b) Using method of variation of parameters, solve

   $$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$ **(7½)**

4. (a) Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ **(7½)**

   (b) Prove that $e^{x \sin x} = 1 + x^2 + \frac{x^4}{3} + \frac{x^6}{120} + ...$ **(7½)**

SECTION - C

5. (a) Investigate the values of λ and μ so that the equations

   $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have

   (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. **(7½)**

   (b) Show that the eigen values of a unitary matrix have the absolute value 1. **(7½)**

6. (a) Using the Gauss-Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ **(7½)**

   (b) Factorize the matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ into LU, where L is lower triangular matrix and U is the upper triangular matrix. **(7½)**

SECTION - D

7. (a) Determine which of the following functions are analytic :

(i) $\frac{1}{z}$ (ii) $\frac{x+iy}{x^2+y^2}$ (iii) $\frac{1}{2}\log\left(x^2+y^2\right)+i\tan^{-1}\frac{y}{x}$. **(7½)**

(b) Show that the polar form of Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta},\ \frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}.$$ **(7½)**

8. (a) Find the general value of θ which satisfies the equation

$[\cos\theta + i\sin\theta]\,[\cos 2\theta + i\sin 2\theta]....[\cos n\theta + i\sin n\theta] = 1.$ **(7½)**

(b) Sum the series $\sin\alpha + x\sin(\alpha+\beta)+\frac{x^2}{2!}\sin(\alpha+2\beta)+......\infty$ **(7½)**

SECTION - E

9. (a) Evaluate the integral $\int_1^2\int_1^3 xy^2\,dx\,dy.$

(b) Change the order of integration in the integral

$$I=\int_{-a}^{a}\int_{0}^{\sqrt{(a^2-y^2)}} f(x,y)\,dx\,dy.$$

(c) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz.$

(d) Solve the following linear differential equation:

$$\frac{d^2x}{dt^2}+3a\frac{dx}{dt}-4a^2x=0.$$

(e) Prove that $\tan x = x+\frac{x^3}{3}+\frac{2}{15}x^5+........$

[P.T.O.]

(f) Show that the diagonal elements of a skew-Hermitian matrix must be pure imaginary numbers or zero.

(g) If λ is an eigen values of an orthogonal matrix, then show that $\frac{1}{\lambda}$ is also its eigen value.

(h) Find the product of the eigen values of $\begin{bmatrix} 7 & 2 & 2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

(i) Prove that $\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5} = 1.$

(j) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\frac{b}{a}$. Hence, evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$.

(4×10=40)