[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2123)

1508

MCA 2nd Semester Examination

Discrete Mathematics

MCA-203

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all selecting one question each of Sections A, B, C and D. Question no. 9 in Section E is compulsory.

SECTION - A

1. (a) Prove the equivalence:

$7(P \wedge Q) \rightarrow (7P \vee (7P \vee Q)) \Leftrightarrow (7P \vee Q)$ **(6)**

(b) Show that $P \wedge (7Q \vee R)$ and $P \vee (Q \wedge 7R)$ are logically not equivalent. **(6)**

2. (a) Obtain the principal conjunctive and disjunctive normal forms of

$(7P \vee 7Q) \rightarrow (P \rightleftarrows 7Q)$ **(6)**

(b) Establish the validity of the following argument:

$p \rightarrow (q \rightarrow r)$
$p \vee 7s$
q
―――――
$\therefore s \rightarrow r$ **(6)**

SECTION - B

3. (a) Let x be a non-empty set. Let P be the class of all subsets of x. For A and B in P consider the relation $A \leq B$ if $A \subseteq B$, show that P is a lattice w.r.t. , $\leq$, relation. **(6)**

(b) Let (P, $\leq$) be a distributive lattice, show that, if

$a \wedge x = a \wedge y$ and $a \vee x = a \vee y$

for some a, then x = y. **(6)**

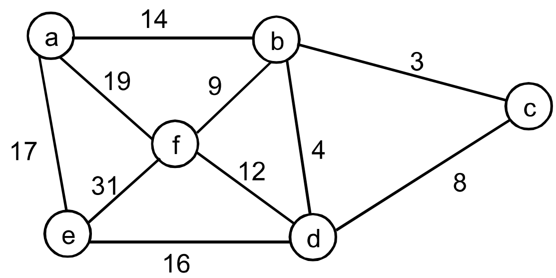
4. (a) For any a and b in a Boolean algebra prove that

$\overline{a \vee b} = \bar{a} \wedge \bar{b}$ and $\overline{a \wedge b} = \bar{a} \vee \bar{b}$. **(6)**

(b) Define an equivalence relation. Construct an example of a relation which in an equivalence relation. Also give an example of a relation which is not an equivalence relation. Justify your answer. **(6)**

SECTION - C

5. (a) Find the minimum spanning tree for the weighted graph given below: **(6)**



(b) Define the terms Eulerian path, Hamiltonian path and planar graph. Give an example of each. **(6)**

6. (a) Show that an undirected graph possesses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree. **(6)**

(b) Prove the relation

$C(n, r) = C(n-1, r) + C(n-1, r-1)$

by using generating functions. **(6)**

SECTION - D

7. (a) Find all solutions of the recurrence relation

$$a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3n.$$ **(6)**

(b) State and prove Lagrange's theorem. **(6)**

8. (a) Let A be the set of binary sequences of length n. Let $\oplus$ be a binary operation of A & t. For x and y is A, $x \oplus y$ is a sequence of length n that has 1∧s in those positions x and y differ and has o∧s in these positions x and y are the same. Show that (A, $\oplus$) is a group. **(6)**

(b) Let R be the group of real numbers w.r.t. the addition operation and R_+ be the group of positive real numbers w.r.t. multiplication operation. Prove that the mapping

$f : R \to R_+$

defined by f(x) = e^x for $x \in R$ is an isomorphism. **(6)**

SECTION - E

9. (i) Construct truth table for $7Q \vee (P \wedge Q)$.

(ii) Write in symbolic form. If either Ram takes History or Radha takes Hindi then Shyam shall take Computer Science.

(iii) Give an example of a relation on a set which is not a partial order relation.

(iv) What is meant by a functionally complete set of connectives? Give an example also.

(v) What is the difference between a relation and a function?

(vi) What is the difference between an integral domain and a field?

[P.T.O.]

(vii) Give an example of a finite group and an infinite group.

(viii) Give an example of a non-planar graph.

(ix) Give an example of a nth order linear recurrence relation.

(x) Define a normal subgroup. Can a group have all its subgroup as normal subgroups?

(xi) Explain the idea of cut-sets in graphs.

(xii) Give an example of a ring. **(1×12=12)**