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B. Tech 3rd Semester Examination

Engineering Mathematics (N.S.)

NS-206

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt one question from each section A, B, C and D. Section E is compulsory. All questions carry equal marks.

SECTION - A

1. (a) Form the partial differential equation corresponding to $F(xy + z^2, x + y + z) = 0$

   (b) Find the solution of differential equation $x^2p + y^2q = (x + y)z$

   (c) Find the complete integral of $p^2 - y^2q = y^2 - x^2$

   (d) Find the solution of $x^2p^2 + y^2q^2 = z^2$ **(20)**

2. (a) Solve $\left(D_x^2 + 3D_xD_y + D_x + 2D_y^2 - 2\right) z = e^{3x+4y} + y(1-2x)$

   (b) Find the solutions of one dimensional heat conduction equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by using the method of separation of variables. Hence find the solutions satisfying the boundary conditions u(0,t) = 0 = u(L,t) where 'L' is length of rod, and initial condition u(x,0) = f(x). **(20)**

SECTION - B

3. (a) Find the solution of $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ using power series method. Also find its radius of convergence.

   (b) State and prove the orthogonality of Legendre functions.

(20)

4. (a) Define generating function. Hence prove that function $\exp\left\{\frac{1}{2}x\left(t-\frac{1}{t}\right)\right\}$ is generating function for Bessel functions.

   (b) Solve the initial value problem

   $x\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0,\ y(1) = 2,\ y'(1) = 4$ **(20)**

SECTION - C

5. (a) Find the solution of IVP $y'' + 2y' + 5y = e^{-t}\sin t$, $y(0) = 3$, $y'(0) = 1$ by using Laplace transform.

   (b) Find the solution of $\frac{d^4y}{dx^4} = \frac{W}{EI}\delta\left(x-\frac{L}{3}\right)$ such that

   $y(0) = 0 = y'(0)$ and $y(L) = 0 = y'(L)$ **(20)**

6. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{if } |x| \le a \\ 0 & \text{otherwise} \end{cases}$

   Hence evaluate $\int_0^\infty \frac{\sin ax}{x}dx$

   (b) Find the inverse Laplace transform of

   $f(s) = \frac{s^3 - 3s^2 + 6s - 4}{(s^2 - 2s + 2)^2}$ **(20)**

SECTION - D

7. (a) If f(z) is analytic function show that it satisfies Cauchy Riemann Equations. Hence show that real and imaginary parts of analytic function are harmonic.

(b) Determine F(2), F(4), F′(i) if $F(\alpha)=\oint_C \frac{5z^2-4z+3}{z-\alpha}dz$

where C is the ellipse $16x^2 + 9y^2 = 144$ **(20)**

8. (a) Evaluate the following using complex variable technique

(i) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ (ii) $\int_0^{\infty} \frac{\cos mx}{x^2+1}dx$ (iii) $\int_0^{\pi} \frac{a\,d\theta}{a^2+\sin^2\theta}$ **(20)**

SECTION - E

9. (a) Prove that z = 0 is removable singularity of $\frac{z-\sin z}{z^2}$

(b) Find the complementary function of

$\left(D_x^3 - 3D_x^2D_y + 2D_y^2D_x\right)z = 0$

(c) Find the complete solution of (p + q) (z – xp – yq) = 1

(d) Find the complete solution of p – q = In(x + y)

(e) What do you mean by Regular Singular Point.

(f) State Cauchy Integral formula.

(g) If L{f(t)} = F(s) then prove that $L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

(h) Find the Fourier transform of $f(x)=\begin{cases}\frac{1}{2a} \text{ if } |x| \le a \\ 0 \quad \text{otherwise}\end{cases}$

(i) Prove that $\frac{d}{dx}x^pJ_p(x) = x^pJ_{p-1}(x)$

(j) Using substitution reduce differential equation $x^2y'' + xy' + (\lambda^2x^2 - p^2)y = 0$ into Bessel's differential equation.

(2×10=20)