[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2123)

1301

B. Tech 1st Semester Examination

Engineering Mathematics-I (N.S.)

NS-101

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each section A, B, C & D of the question paper and all the subparts of the question in section E.

SECTION - A

1. (a) For what values of k the equations x + y + z = 1, 2x + y+4z = k, 4x + y + 10z = k^2 have a solution and solve them completely in each case.

   (b) Find the characteristic values and characteristic vectors of $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **(10+10=20)**

2. (a) Define Skew Hermitian Matrix and show that the eigen values of skew-hermitian matrix is either zero or purely imaginary.

   (b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 2yz + 4zx - 4xy$ to a canonical form. Also write the model matrix and nature of the quadratic form. **(10+10=20)**

SECTION - B

3. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

(b) Examine $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum values. **(10+10=20)**

4. (a) If $z = \sin^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ then find the value of

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2}.$$

(b) Find the point upon the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value. Also find this minimum value. **(10+10=20)**

SECTION - C

5. (a) Evaluate the following integral by changing the order of integration

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx.$$

(b) Find, by triple integration, the volume bounded above, by the sphere $x^2 + y^2 + z^2 = 2a^2$ and bounded below by the paraboloid $az = x^2 + y^2$. **(10+10=20)**

6. (a) Using the concept of double integrals, evaluate $\iint_R (x+y)^2\,dxdy$, where R is the region bounded by parallelogram $x + y = 0$, $x + y = 2$, $3x - 2y = 0$, $3x - 2y = 3$.

(b) Evaluate the following integral by changing to spherical polar coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\, dz\, dy\, dx.$$ **(10+10=20)**

SECTION - D

7. (a) Separate real and imaginary parts of $\sin^{-1}(\cos\theta + i\sin\theta)$, where θ is a positive acute angle.

(b) Sum the series

$\cos\alpha + x\cos(\alpha+\beta) + \frac{x^2}{2!}\cos(\alpha+2\beta) + \ldots\infty$

(10+10=20)

8. (a) If $\tan(\theta+\phi) = e^{i\alpha}$, show that

$$\theta = \left(n+\frac{1}{2}\right)\frac{\pi}{2} \text{ and } \phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right).$$

(b) Use De Moivre's theorem to solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. **(10+10=20)**

SECTION - E

9. (a) State and prove Euler's theorem for homogenous function.

(b) Separate $\log(l + i\tan\alpha)$ into real and imaginary parts.

(c) If $x^2 + y^2 + 3axy = c$, Find $\frac{dy}{dx}$.

(d) Use Maclaruin series to prove $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

(e) Show that $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{y^2-x^2}{y^2+x^2}$ does not exist.

[P.T.O.]

(f) Find the eigen values of the matrix $A = \begin{bmatrix} 1+i & -6 \\ 8 & 3-5i \end{bmatrix}$.

(g) Define linear dependent and linear independent vectors.

(h) If x = r cosθ, y = r sinθ, then verify that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$

(i) Evaluate the integral $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$.

(j) Discuss the C+iS method for summation of a sine and cosine series. **(10×2=20)**