

MCA 2nd Semester Examination
Discrete Mathematics (NS)
MCA-203

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all selecting one question from each sections A, B, C and D. Section E is compulsory.

SECTION - A

1. (a) Show that $[s \rightarrow (((\neg p) \wedge q) \wedge r)] \Leftrightarrow \neg[(p \vee (\neg(q \wedge r))) \wedge s]$.
(6)

(b) Show that " $(p \wedge q) \rightarrow (p \vee q)$ " is a tautology, while " $((\neg p) \wedge q) \wedge (p \vee (\neg q))$ " is a contradiction. (6)

2. (a) Establish the validity of the argument

$$\begin{array}{l} p \rightarrow q \\ (\neg r) \vee (\neg q) \\ r \\ \hline \neg p \end{array} \quad (6)$$

(b) Check, is the statement $((p \wedge q) \vee r) \wedge \rightarrow ((p \wedge q) \vee (\neg q))$ is in disjunctive normal form. (6)

[P.T.O.]

SECTION - B

3. (a) Let (L, \leq) be a lattice in which '.' And '+' denote the operation of meet and join respectively. Then

$$b \leq c = \begin{cases} a.b \leq a.c \\ a + b \leq a + c \end{cases} \quad \forall a, b, c \in L \quad (6)$$

(b) Define complemented lattice. Show that lattice $(L_3, <_3)$ of 3-tuple of 0 and 1 is a complemented lattice. (6)

4. (a) Simplify the Boolean expression

$$Y = \overline{A}B\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + AB\overline{C} \quad (6)$$

(b) Construct NAND gate structure for the expression

$$Z = (\overline{A} + B)C + \overline{F} + DE \quad (6)$$

SECTION - C

5. (a) Prove that connected graph has a spanning tree. (6)

(b) Define Minimal Spanning Tree. Show that minimum height of a binary tree on n vertices is $\lceil \log_2(n+1) - 1 \rceil$. (6)

6. (a) A connected multi graph is Eulerian iff its edge set can be partitioned into cycles. (6)

(b) Explain Multi-graphs and weighted graphs with examples. (6)

SECTION - D

7. (a) Let $(G, *)$ be a group and H is a non empty subset of G, then H is subgroup of G if and only if $a, b \in H \Rightarrow a * b^{-1} \in H$. (6)

(b) A sub group H of group G is normal in G iff $gH = Hg$ $\forall g \in G$. (6)

8. (a) Using the method of generating function solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$, $n \geq 2$, given that $a_0 = 3$, $a_1 = -2$. (6)
- (b) Define ring. If Q be the set of all rational numbers and '+' and '.' be two binary operations. Then discuss is $(Q, +, \cdot)$ a ring or not? (6)

SECTION - E

9. (a) Obtain conjunctive normal form of $p \rightarrow (q \wedge r)$. (2)
- (b) Construct truth table for conditional statement $p \rightarrow q$. (1)
- (c) Define the terms (i) Partially Ordered Set (ii) Linearly Ordered Set with examples. (1)
- (d) Define consensus method to simplify Boolean expression. The consensus of AB and $A'C$ is _____. (1)
- (e) Define Cut Set and Cut Edges with examples. (1)
- (f) Compare and define Euler Path and Hamiltonian Path. (1)
- (g) Show that $G = \{1, -1, i, -i\}$, where $i = \sqrt{-1}$, is an abelian group with respect to multiplication as a binary operation. (2)
- (h) Define field, illustrate with example. (1)
- (i) Solve the recurrence relation $a_r - 7a_{r-1} + 12a_{r-2} = 1$. (1)
- (j) Define Planar and non-planar graphs with examples. (1)