[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2126)

 16100(D)

B. Tech 3rd Semester Examination

Signals and Systems (CBS)

EC-304

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each of the section A, B, C and D. Section E (question 9) is compulsory.

SECTION - A

1. (a) Consider the cascade of the following two systems S_1 and S_2 as depicted in Figure 1.

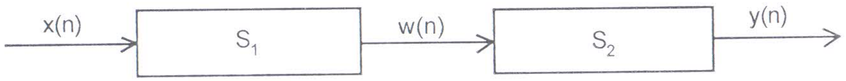

Figure 1

S_1: causal LTI : $w(n) = \frac{1}{2}w(n-1) + x(n)$

S_2: causal LTI : $y(n) = \alpha y(n-1) + \beta w(n)$

The difference equation relating x(n) and y(n) is:

$y(n) = -\frac{1}{8}y(n-2) + \frac{3}{4}y(n-1) + x(n)$. Determine α and β

(b) Find the fundamental period of the discrete time signal $x[n] = 1 + e^{j2\pi n/3} - e^{j4\pi n/7}$ (5+5=10)

2. (a) Show that a relaxed linear time invariant system is causal if and only if h[n]=0, for n<0.

(b) Find the fundamental period of the continuous time signal

$x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t-2k)$. (5+5=10)

SECTION - B

3. (a) The impulse response of the system is $h(t)=te^{-at}u(t)$. Find the unit step response of the system.

(b) State and prove the Parseval's relation for continuous-time periodic signal. (5+5=10)

4. (a) Evaluate the unit step response for the LTI system represented by the impulse response $h[n] = \delta(n)-\delta(n-2)$.

(b) If $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental period T_0 and their complex Fourier series expressions are

$x_1(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ and $x_2(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$, where

$\omega_0 = \frac{2\pi}{T_0}$ show that the signal $x(t)=x_1(t)\,x_2(t)$ is periodic with the same fundamental period T_0 and can be expressed as $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ where $c_k = \sum_{m=-\infty}^{\infty} a_m b_{k-m}$.

(5+5=10)

SECTION - C

5. (a) Find the final value of the response y(t) of a system whose transfer function is $H(s) = \frac{s+3}{s^2+4s+5}$ when the system is excited by a unit step, i.e , x(t)=u(t).

(b) Find the Fourier transform of the signum function

$$x(t) = \mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$ (5+5=10)

[P.T.O.]

6. (a) Find the inverse Laplace transform of $X(s)=\ln\left(1+\frac{\omega^2}{s^2}\right)$

(b) Find the Fourier transform of the unit step function

$$x(t)=u(t)=\begin{cases}1, & t\geq 0\\ 0, & t<0\end{cases}$$ (5+5=10)

SECTION - D

7. (a) A causal discrete-time LTI system is described by $y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)$ where x(n) and y(n) are the input and output of the system, respectively. Find the step response of the system using z-transform method.

(b) Explain frequency aliasing in sampling process. Show that the sampled version of the signals $x_1(t)$ = 10cos(100πt) and $x_2(t)$ = 10cos(50πt) are identical when sampled with sampling frequency f_s=75 Hz. (5+5=10)

8. (a) Apply the final value theorem to determine $x(\infty)$ for the signal $x(n)=\begin{cases}1, & \text{if n is even}\\ 0, & \text{otherwise}\end{cases}$

(b) Define the sampling theorem. Determine the Nyquist rate of the signal x(t)=sin c(200πt)+sin c^2(200πt). (5+5=10)

SECTION - E

9. (a) Find the even and odd components of $x(t)=e^{jt}$.

(b) Show that if x(t) is even then $\int_{-\alpha}^{\alpha} x(t)dt = 2\int_{0}^{\alpha} x(t)dt$.

(c) Show that the complex exponential sequence $x[n]=e^{j\omega_0 n}$ is periodic only if $\omega_0/2\pi$ is a rational number.

(d) Determine whether the signal $x[n]=\cos^2\left(\frac{\pi}{8}n\right)$ is periodic. If yes then determine the fundamental period.

(e) Prove the validity of the equality $\sum_{n=0}^{N-1} a^n=\begin{cases}\frac{1-a^n}{1-a} & a\neq 1\\ N & a=1\end{cases}$

(f) Compute the average power of the signal x[n] = u[n].

(g) Obtain the relationship between Laplace transform and continuous time Fourier transform.

(h) Consider a discrete-time LTI system with impulse response $h[n]=a^n u[n]$. Determine under what condition the system is BIBO stable and unstable.

(i) A system is described by the input output relationship as $y(n)=\sum_{k=-\infty}^{n} x(k)$. Find its inverse system.

(j) List the Dirichlet conditions. (10×2=20)