

16349(D)

M. Tech 3rd Semester Examination

Advanced Mathematics

EC-306

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt any five questions.

1. (a) Define Sturm-Liouville equation. Give example. (4)
- (b) Find the eigen values and eigen functions of the boundary value problem:
$$y''(x) + \lambda y(x) = 0 \quad : 0 < x < \pi$$
given that $y(0) = y(\pi) = 0$ (8)
- (c) Define orthogonal functions. Prove that eigen functions corresponding to distinct eigen values of a regular Sturm-Liouville boundary value problem are orthogonal. (8)
2. (a) Define bilinear transformation. Construct the bilinear transformation $w=S(z)$, that maps the points $z_1=-\pi, z_2=1, z_3=\pi$ onto the points $w_1=-1, w_2=0, w_3=1$ respectively. (10)
- (b) Define the following:
 - (i) Positive definite
 - (ii) Positive semi definite
 - (iii) Negative definite
 - (iv) Negative semi definite
 - (v) Indefinite

(vi) Range of quadratic form.

Check for definiteness of the following matrix:

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 5 \\ 3 & 0 & 4 & 0 \\ 0 & 5 & 0 & 6 \end{bmatrix} \quad (10)$$

3. (a) Determine the nature of eigen values of Hermitian matrix. (5)
- (b) Find the eigen values of the matrix
$$\begin{bmatrix} 6 & 0 & 9 \\ 0 & 2 & 3 \\ 9 & 3 & 5 \end{bmatrix} \quad (5)$$
- (c) Solve the following system of linear equations by Gauss elimination method:
$$\begin{aligned} x_1 + 9x_2 + x_3 + x_4 &= 12 \\ 7x_1 + x_2 + x_3 + x_4 &= 10 \\ x_1 + x_2 + 6x_3 + x_4 &= 9 \\ x_1 + x_2 + x_3 + 10x_4 &= 13 \end{aligned} \quad (10)$$
4. (a) Find the principal value of $\log(-5i)$. (6)
- (b) Let $x=(1+i)^{1/5}$, then find all the values of x . (7)
- (c) Prove that:
$$\sin 47 + \sin 61 - \sin 11 - \sin 25 = \cos 7 \quad (7)$$
5. (a) By using power series method, prove that the solution of the differential equation:
$$y'' + 4y = 0$$
is $y(t) = A \sin 2t + B \cos 2t$
where $A = \frac{y'(0)}{2}$ and $B = y(0)$ (8)

(b) Prove that:

$$x^6 = \frac{1}{231} [33P_0(x) + 110P_2(x) + 72P_4(x) + 16P_6(x)]$$

where $P_n(x)$ are Legendre's polynomials. (8)

(c) Write a note on modified Bessel's function. (4)

6. (a) Use Frobenius method to solve the differential equation:

$$x^2 y''(x) + xy'(x) + (x^2 - 1)y(x) = 0 \quad (10)$$

(b) Prove that:

$$\cos x = J_0(x) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(x)$$

$$\text{and } \sin x = 2 \sum_{n=0}^{\infty} (-1)^n J_{2n+1}(x) \quad (10)$$

7. (a) Using Jacobi's method, solve the system of equations:

$$5x - y + z = 10$$

$$2x + 4y = 12$$

$$x + y + 5z = -1; \text{ Start with the solution } (2, 3, 0) \quad (8)$$

(b) Solve the following:

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

by Crout's triangularization method. (12)

8. (a) Using any iteration method, find all the eigen values and eigen vectors of the matrix.

$$\begin{bmatrix} 7 & 0 & 9 \\ 0 & 5 & 3 \\ 9 & 3 & 2 \end{bmatrix} \quad (10)$$

(b) Explain Gauss-Seidal iteration method. Why this method is faster than Jacobi's iteration method and how much faster? (10)