[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
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16005(D) - 0 DEC 2016

B. Tech 1st Semester Examination

Engineering Mathematics-I (CBS)

MA-101

Time : 3 Hours Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : A candidate is required to attempt five questions in all, selecting one question from each unit and Question No. 9 is compulsory.

UNIT - I

1. (a) For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. Find non-singular matrices P and Q such that PAQ is in the normal form. (6)

(b) Investigate for what values of λ and μ the simultaneous equations
$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have
(i) no solution, (ii) a unique solution (iii) an infinite number of solutions. (6)

2. (a) If λ be an eigen value of a non-singular matrix A. Show that $\frac{|A|}{\lambda}$ is an eigen value of matrix adj. A. (6)

(b) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence find the matrix represented by
$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + 1$. (6)

UNIT - II

3. (a) Find all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$. Also show that the continued product of these values is 1. (6)

(b) Sum the series by C+iS method
$\sin\alpha + x\sin(\alpha+\beta) + \frac{x^2}{2!}\sin(\alpha+2\beta) + ...\infty$. (6)

4. (a) If f(z) be an analytic function of z, show that
$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0$. (6)

(b) Find the analytic function, whose imaginary part is $\frac{(x-y)}{(x^2+y^2)}$. (6)

UNIT - III

5. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (6)

(b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ hy using Lagrange's Method of undetermined multipliers. (6)

6. (a) Find the volume (by double integrals) bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (6)

(b) Evaluate the following integral by changing to spherical polar co-ordinates:
$$\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} \frac{dx\,dy\,dz}{\sqrt{(1-x^2-y^2-z^2)}}$$ (6)

[P.T.O.]

UNIT - IV

7. (a) Find the unit vector normal to the surface $xy^3z^2=4$, at the point $(-1, -1, 2)$. (6)

(b) Show that $\nabla^2\left(r^n\right) = n(n+1)r^{n-2}$. (6)

8. (a) If $F = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C F.dR$, where C is the curve in the xy-plane $y = 2x^2$ from (0. 0) to (1,2). (6)

(b) Evaluate by Stoke's theorem $\int_C F.dR$ where $F = y\hat{i} + xz^3\hat{j} - zy^3\hat{K}$, C is the circle $x^2 + y^2 = 4$. $z = 1.5$. (6)

UNIT - V (Compulsory)

9. Each part of this question carry one mark.

(a) Define eigen values and eigen vectors of a matrix.

(b) Show that every diagonal element of a Hermitian matrix must be real.

(c) How many distinct values of $(\text{cis}\theta)^{6/15}$?

(d) Write down the polar form of Cauchy-Riemann equations.

(e) State Euler's theorem.

(f) If u and v are functions of two independent variables x and y, then define Jacobian.

(g) Evaluate the integral $\int_1^2\int_1^3 xy^2dxdy$.

(h) Define Beta function.

(i) What is the directional derivative of a scalar point function ϕ along the direction of unit vector a?

(j) If $R = x\hat{i} + y\hat{j} + z\hat{K}$, show that $\nabla.R = 3$.

(k) State Stoke's theorem.

(l) Using divergence theorem, prove that $\int_S R.dS = 3V$.

(12×1=12)