[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
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B. Tech 2nd Semester Examination

Engineering Mathematics-II (CBS)

MA-202

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all selecting one question from each unit. Question no. 9 is compulsory.

UNIT - I

1. (a) Solve the differential equation:

$$(xy^3 + y)dx + 2(x^2y^2 + x + y^4)\,dy = 0 \quad (4)$$

(b) Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$ (4)

(c) Solve $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ (4)

2. (a) Solve by method of variation of parameters:

$$\frac{d^2y}{dx^2} + 4y = \tan 2x \quad (6)$$

(b) A cup of coffee at temperature 100°C is placed in a room of temperature 15°C and it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes. (6)

[P.T.O.]

UNIT - II

3. (a) Use the method of Frobenius to find the solution of the differential equation

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

in the interval $0 < x < R$ (6)

(b) Using generating function of Bessel functions show that

$$J_n(x+y) = \sum_{r=-\infty}^{\infty} J_r(x)J_{n-r}(y) \quad (6)$$

4. (a) Find the solution of differential equation

$$\left(1-x^2\right)y'' - 2xy' + n(n+1)y = 0$$

and define the solution. (6)

(b) Show that for the legendre polynomial

$$\int_{-1}^{1} P_n(x)P_m(x) = \frac{2}{2n+1}\delta_{mn}.$$

where δ_{mn} is the Kronecker delta. (6)

UNIT - III

5. (a) Find the Laplace transform of

(i) $\frac{e^{-t}\sin t}{t}$ (ii) $\frac{1-\cos 2t}{t}$ (6)

(b) Find the inverse Laplace transform of

$$\log\left(\frac{1+s}{s}\right) \quad (6)$$

6. (a) Solve by using transform method:

$$y'' + 4y' + 3y = e^{-t}$$

$$y(0) = 1 \qquad y'(0) = 1$$ (6)

(b) Find the Laplace transform of periodic function

$$f(t) = t \qquad 0 < t < c$$

$$= 2c - t \qquad c < t < 2c$$ (6)

UNIT - IV

7. (a) Find the Fourier series for the function f(x)

$$f(x) = 0 \qquad -\pi \le x \le 0$$

$$= \sin x \qquad 0 \le x \le \pi$$

Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots = \frac{1}{2}$ (6)

(b) State and prove Parseval's identity. (6)

8. (a) Solve the PDE

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x - y$$ (6)

(b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = e^{x+y}$ (6)

UNIT - V

9. (i) Write the Cauchy's linear differential equation.

(ii) State the conditions for the existence of Laplace transform of a function f(t).

(iii) Express $J_1'(x)$ in terms of $J_0(x)$ and $J_2(x)$.

(iv) State the convolution property of Laplace transform.

(v) State the Dirichlet's condition for Fourier series.

(vi) Define complementary function of a differential equation.

(vii) Draw the graph of Heaviside unit step function.

(viii) Write the Laplace transform of $t^{1/2}$.

(ix) Express $Y_n(x)$ in terms of $J_n(x)$ and $J_{-n}(x)$.

(x) Write the expression for $P_n(1)$.

(xi) Define partial differential equation of order k.

(xii) State the necessary and sufficient condition for the differential equation Mdx + Ndy = 0 to be exact.

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